

The Time Path of Scarcity Rent in the Theory of Exhaustible Resources

Y. H. Farzin

The Economic Journal, Vol. 102, No. 413 (Jul., 1992), 813-830.

Stable URL:

<http://links.jstor.org/sici?sici=0013-0133%28199207%29102%3A413%3C813%3ATTPOSR%3E2.0.CO%3B2-K>

The Economic Journal is currently published by Royal Economic Society.

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/res.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is an independent not-for-profit organization dedicated to creating and preserving a digital archive of scholarly journals. For more information regarding JSTOR, please contact support@jstor.org.



THE TIME PATH OF SCARCITY RENT IN THE THEORY OF EXHAUSTIBLE RESOURCES*

Y. H. Farzin

There is now a consensus in the literature on resource economics that a depletable resource should command a price in excess of its marginal extraction cost, simply to reflect the opportunity cost of using a unit of it today rather than conserving it for the future. But how this excess is to change along an optimal (or, competitive equilibrium)¹ path of resource use is still an unresolved question. This might at first seem puzzling, since according to the fundamental theorem of economics of exhaustible resources, or Hotelling (1931) rule, on an optimal path, price minus marginal extraction cost (also termed interchangeably as 'scarcity rent', 'user cost', or 'royalty') should steadily grow over time at the social rate of discount. However, the literature contains sharply contrasting views on the subject. For example, Kay and Mirrlees (1975) have argued that since along an optimal path the scarcity rent grows exponentially as a finite stock of a resource nears exhaustion, then situations in which the resource stock is still very large and yet the price greatly exceeds the marginal extraction cost should indicate that the price is significantly above its optimal level and that the resource is overconserved.

In stark contrast to this view, Heal (1976) has argued that as the resource stock nears exhaustion the difference between price and marginal cost (or, what he termed, 'social cost of extraction') falls monotonically to zero.

Hanson (1980) analyses the same model as Heal's, but unlike Heal who assumes the unit extraction cost to be convex, he postulates it to be concave, and yet, like Heal, concludes that the scarcity rent will be monotonically declining over time.

Solow and Wan (1976) analyse a still different situation where resource deposits differ in quality and the unit extraction cost increases as higher-grade deposits are exhausted and extraction proceeds to lower grades. They show that along an optimal path the shadow price of a resource must rise at the real rate of interest despite differential extraction costs, *but* that the difference between price and the marginal extraction cost, or what they call 'degradation charge' declines monotonically over time to zero.

These sharply differing conclusions suggest that, far from being resolved, the question of how scarcity rent changes over time is much in want of a close scrutiny. The need for such a scrutiny becomes even stronger once it is noted

* I wish to thank Robert Solow, James Mirrlees, Pierre Lasserre, and anonymous referees of this JOURNAL for very helpful discussions and comments on earlier drafts. A previous version of this paper was presented at the Non-Renewable Resources session of the Allied Social Science Association annual meeting in Washington, D.C., December 28-30, 1990.

¹ In view of the basic theorem of welfare economics, the terms 'optimal' and 'competitive equilibrium' are used interchangeably.

that in addition to being at variance with each other, the theoretical judgments have been also vigorously contested by empirical findings. Notable in this regard has been the pioneering work of Barnett and Morse (1963) who found that unit costs for most extractive industries declined in real terms during the period 1870 to 1957, leading them to conclude that rather than growing more scarce, extractive resources became more abundant. The updates of this work to the early 1970s by Barnett (1979) and Johnson *et al.* (1980) reached the same general conclusion. Another challenge to the hypothesis of steadily increasing resource scarcity has come from Nordhaus' (1974) finding that the real prices for 11 major minerals fell substantially over the period 1900 to 1970.

Of course, empirical studies themselves have differed vastly in their conclusions regarding the direction of resource scarcity trend. For example, Smith (1979a), Slade (1982) and Hall and Hall (1984) have analysed time trends in real prices for many natural resources and, in contrast to the empirical studies just mentioned, have found that in general the resource scarcity has been moving on a rising, not declining, trend.

Aside from disparities in data definition, time coverage, and application method, a potential source of these conflicts has been a lack of consensus among economists regarding an appropriate measure of scarcity. As Brown and Field (1978) and Fisher (1979) have noted, as measures of resource scarcity, both the unit cost and price of extracted resource suffer from conceptual shortcomings. On the other hand, the shadow scarcity rent which seems to have been preferred as a true economic indicator of scarcity by most economists (among them Pindyck (1978), Brown and Field (1978), Halvorsen and Smith (1984)) has its own practical limitation of not being directly observable.² This in part explains why only few empirical studies have been conducted in relation to the scarcity rent (notable examples are Farrow (1985), Halvorsen and Smith (1984), Pesaran (1990), and Lasserre and Ouellette (1991)).

However, for the most part, the disagreements both among the empirical findings and between them and theoretical judgments stem from limitations of particular models on which the theoretical views on the dynamics of scarcity rent have rested. Therefore, as a first step toward reconciliation of these conflicts, in this paper I scrutinise the scarcity rent within a model of exhaustible resource extraction to yield a deeper theoretical insight into its behaviour over time than has yet been provided in the literature. In the next section, I derive general conditions characterising the time path of scarcity rent under competitive conditions. I postulate a general extraction cost function that allows for the effects of resource depletion and technological change as well as that of extraction rate. It is shown that the time path of scarcity rent is more complicated than usually understood in that, contrary to the views in the existing literature, it is generally *non-monotonic*. In fact, it is shown in Section II.A that the theoretical conclusions in the literature, including the conventional view that the scarcity rent for an exhaustible resource should always increase over time, either hold only as special cases of the general result

² Also see Fisher (1981), Devarajan and Fisher (1982) and Lasserre (1985) for a discussion of the marginal discovery cost as a proxy for the shadow scarcity rent.

obtained in this paper, or in some cases are not necessarily valid. An interesting corollary derived here shows that when the discount rate is zero (as when the consideration of intergenerational equity in the sense of Rawls' maxi-min criterion governs resource exploitation) then, contrary to Hotelling rule, the scarcity rent always *declines* over time monotonically to zero, *irrespective* of the form of extraction costs function. In light of this corollary, the scarcity rent path characterised by Solow and Wan (1976) is found to present a highly special case. Furthermore, it is argued in Section II.B that the hypothesis of non-monotonic scarcity rent advanced here is general enough to reconcile the empirical findings that otherwise appear to have been in defiance of the conventional theory or in conflict with each other. In Section III it is shown that except for extremely special situations, technological change generally reinforces the non-monotonic behaviour of the scarcity rent path. Concluding remarks are presented in Section IV.

I. THE MODEL

Consider a typical competitive owner of an exhaustible resource who can obtain the market price p_t for the resource at time t . His total extraction cost at time t is given by a twice continuously differentiable function $C_t = C(x_t, X_t, z_t)$, where x_t is the rate of extraction at time t , $X_t = \int_0^t x_\tau d\tau$ is the cumulative amount extracted up to time t , and z_t is an index of the state of technology at time t , with $\dot{z}_t > 0$ measuring its incremental improvement over time. For a given state of technology, the total extraction cost increases both with the current extraction rate (i.e. $C_{x_t} > 0$) and the cumulative extraction up to date (i.e. $C_{X_t} > 0$), but, other things equal, it decreases as a result of technological progress (i.e. $C_{z_t} < 0$).³

Furthermore, in view of geological and engineering knowledge about exploitation of depletable resources, one expects the marginal extraction cost C_x to have the familiar properties that $C_{xx} > 0$, $C_{xX} > 0$, and $C_{xz} < 0$. $C_{xx} > 0$ reflects the possible effect of *diminishing returns* to extraction rate which causes the marginal extraction cost to rise as the extraction rate is speeded up; $C_{xX} > 0$ reflects the *depletion effect* which raises the marginal cost of maintaining a given rate of extraction as increasing amounts of resource are depleted; and $C_{xz} < 0$ captures the potential cost-reducing effect of *technological improvements*, thus countervailing the other two effects. In symmetry with $C_{xX} > 0$, it is natural to assume that $C_{Xx} > 0$. It is also taken that $C_{XX} > 0$, as is often assumed (see, e.g. Zimmerman (1977) and Pindyck (1978)), and that $C_{Xz} < 0$. Accordingly, the incremental cost due to cumulative extraction rises both with the extraction rate and the amount already extracted, but, all else equal, it falls as a result of technological progress. No fixed quantity is assumed for the total availability of the resource, although, as shall be seen later, only a limited total amount will be economically recoverable at any time. This is intuitively plain. For, as the assumption of $C_{XX} > 0$ implies, barring technological change, increasingly

³ z signifies $\partial z / \partial t$, and the subscripts other than t denote the respective partial derivatives. Subscript t is suppressed where not essential.

large quantities of the resource can be exploited only at increasingly high incremental costs, with $C_X \rightarrow \infty$ as $X \rightarrow \infty$. As such, it will never make economic sense to extract infinite amount of the resource; that is, to let $X \rightarrow \infty$. Finally, it is natural to expect $C_X(0, X_t, z_t) > 0$ for any $X_t > 0$ and any state of technology; that is, at any time when resource exploitation is drawn to a close, the incremental cost due to cumulative extraction must have obtained some positive, and probably large, magnitude.

Assuming that the representative resource owner maximises the present discounted sum of his profits, he seeks an extraction rate x_t so as to

$$\text{maximise}_{x_t} \quad V = \int_0^{\infty} e^{-\rho t} [p_t x_t - C(x_t, X_t, z_t)] dt. \quad (1)$$

$$\text{Subject to} \quad \dot{X}_t = x_t, \quad X(0) = 0, \quad (2)$$

$$x_t \geq 0 \text{ is continuous on } t \in [0, \infty), \quad (3)$$

$$x_t \in U, \cup \text{ a fixed non-negative subset of } R^1, \quad (4)$$

where $\rho > 0$ is the market interest rate, which for simplicity is taken to be constant.

A question of primary concern is whether an optimum policy exists at all. This is dealt with in the Appendix where it is shown that in the case of the present problem the assumption of convexity of $C(x, X)$ in (x, X) ensures the existence of an optimum. In what follows, I derive the necessary conditions for solving the optimisation problem and then use these conditions to establish the behaviour of relevant variables, particularly that of the shadow scarcity rent.

Defining the current-value Hamiltonian as $H = p_t x_t - C(x_t, X_t, z_t) + \lambda_t x_t$, the necessary conditions for an interior optimum are

$$\partial H / \partial x_t = p_t - C_{x_t}(x_t, X_t, z_t) + \lambda_t = 0, \quad (5)$$

$$\dot{\lambda}_t - \rho \lambda_t = -\partial H / \partial X_t = C_{X_t}(x_t, X_t, z_t), \quad (6)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t X_t = 0, \quad (7)$$

where $\lambda_t (< 0)$ is the shadow cost associated with the cumulative extraction up to t . The transversality condition (7) holds with complementary slackness, so that for an interior optimum ($x_t \geq 0$) one must have⁴

$$\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t = 0 \quad \text{and} \quad 0 < \lim_{t \rightarrow \infty} X_t = \bar{X} < \infty. \quad (8)$$

Solving the differential equation (6), it is readily seen that λ_t measures the discounted sum of the incremental costs that an additional unit of resource extracted at time t brings about in that period and also spills over into all future periods by raising cumulative extraction levels X_τ ($\tau \geq t$), that is

$$\lambda_t = - \int_t^{\infty} e^{-\rho(\tau-t)} C_{X_\tau}(x_\tau, X_\tau, z_\tau) d\tau. \quad (9)$$

⁴ That on an optimal path the cumulative extraction reaches an upper limit, \bar{X} , as $t \rightarrow \infty$ follows from the assumption of $C_{XX} > 0$; for, otherwise, $C_X \rightarrow \infty$ as $X \rightarrow \infty$, which is obviously non-optimal.

Viewed from a different vantage point, the shadow externality cost λ_t is just the mirror image of the shadow *value* (or competitive price) of an incremental unexploited unit of the resource, or simply the shadow scarcity rent at time t . Accordingly, along an optimal path, the resource price at any time should cover the marginal extraction cost $C_{x_t}(x_t, X_t, z_t)$ plus the shadow scarcity rent $|\lambda_t|$, that is

$$p_t = C_{x_t}(x_t, X_t, z_t) + \int_t^\infty e^{-\rho(r-t)} C_{X_r}(x_r, X_r, z_r) d\tau. \quad (10)$$

Differentiating (5) with respect to time and substituting for $\dot{\lambda}_t$ in (6) yields

$$(d/dt) \ln [p_t - C_{x_t}(x_t, X_t, z_t)] = \rho - C_{X_t}(x_t, X_t, z_t) \left/ \int_t^\infty e^{-\rho(r-t)} C_{X_r}(x_r, X_r, z_r) d\tau \right. \quad (11)$$

Equation (11) presents the optimal rule of resource depletion in a highly general form. It simply states that, on an optimal path, the *scarcity rent changes* at a rate equal to the rate of discount ρ less the percentage ratio of the discounted externality cost attributable to the current period t , that is⁵ $C_{X_t} / \int_t^\infty e^{-\rho(r-t)} C_{X_r} d\tau$. It is important to note that since in general this ratio varies with time, so does the rate at which the scarcity rent changes.

To examine the behaviour of $|\lambda_t|$ over time, note from (9) that

$$\lim_{t \rightarrow \infty} |\lambda_t| = \lim_{t \rightarrow \infty} \int_t^\infty e^{-\rho\tau} C_X(\tau) d\tau / e^{-\rho t} \quad (12)$$

which, on applying L'Hopital's rule, gives

$$\lim_{t \rightarrow \infty} |\lambda_t| = 1/\rho \lim_{t \rightarrow \infty} C_X(t), \quad (13)$$

where $C_X(\tau)$ is a convenient notation for $C_{X_\tau}(x_\tau, X_\tau, z_\tau)$, as is analogously $C_X(t)$.

Also, substituting for λ_t from (9) into (6) and integrating by parts yields, after some manipulation,

$$|\dot{\lambda}_t| = \int_t^\infty e^{-\rho(\tau-t)} \frac{d}{d\tau} C_X(\tau) d\tau, \quad (14)$$

where $(d/d\tau) C_X(\tau) = C_{Xx}(\tau) \dot{x}_\tau + C_{Xx}(\tau) \dot{x}_\tau + C_{Xz}(\tau) \dot{z}(\tau)$. (15)

Equations (9) and (13)–(15) show vividly that the behaviour of $|\lambda_t|$ is more complex than usually understood in the literature in that it depends crucially on the shape of $C_X(\tau)$ and its rate of change with time. More specifically, it is shown here that, depending on how $C_X(\tau)$ varies with time, the shadow scarcity rent not only can increase monotonically over time, it can also monotonically decrease, or remain constant, or more generally change *non-monotonically* over time.

⁵ This generalised Hotelling rule appears to have been first derived by Kay and Mirrlees (1975) in a general model of extraction with fixed resource stock. Levhari and Liviatan (1977) also derive the same rule for the case where the resource stock is fixed, the extraction horizon is finite, and the exhaustion is incomplete. However, neither of these authors provide a general analysis of the dynamics of scarcity rent. In fact, as shall be argued in the next section, the conclusion of Kay and Mirrlees on the time path of scarcity rent holds as a special case, while that of Levhari and Liviatan, for the case they analyse, does not necessarily hold.

First, by taking limit of (6) as $t \rightarrow \infty$ and using (13), one notes that

$$\lim_{t \rightarrow \infty} |\dot{\lambda}_t| = 0. \quad (16)$$

Next, I concentrate on behaviour of $C_X(t)$ along an optimal path. From the continuity of λ_t and $C_X(t)$ on $t \in [0, \infty)$ and by using (13) and (16) one has

$$\lim_{t \rightarrow \infty} \dot{C}_X(t) = \lim_{t \rightarrow \infty} |\dot{\lambda}_t| = 0. \quad (17)$$

Further, the convexity of $C(x, X)$ in (x, X) , together with the transversality condition that on an optimal path $0 < \lim_{t \rightarrow \infty} X_t = \bar{X} < \infty$, implies that $C_X(x_t, X_t)$ has a finite limit as $t \rightarrow \infty$. To specify the value of this limit, we note from the continuity of X_t on $[0, \infty)$ and the fact that $\lim_{t \rightarrow \infty} X_t = \bar{X}$, that $\lim_{t \rightarrow \infty} x_t = \lim_{t \rightarrow \infty} \dot{X} = 0$. It therefore follows that $\lim_{t \rightarrow \infty} C_X(x_t, X_t) = C_X(0, \bar{X})$ and from (13), that

$$\lim_{t \rightarrow \infty} |\lambda_t| = (1/\rho) C_X(0, \bar{X}) > 0. \quad (18)$$

Finally, the convexity of $C(x, X)$ in (x, X) renders the optimal solution pair $(x(t), X(t))$ unique, which in turn implies that the limits of $C_X(t)$ and λ_t are unique.

Now, consider possible general forms that $C_X(t)$ can take. For simplicity of exposition, and without loss of generality, let us temporarily abstract from the effect of technological change by assuming that $\dot{z} = 0$.

Case (a): $C_X(t)$ increases monotonically over time, that is $\dot{C}_X(t) > 0$;⁶ then, by (14), so will $|\lambda_t|$. Thus, for this case, the shadow scarcity rent rises monotonically over time from the initial level $|\lambda_0| = \int_0^\infty e^{-\rho t} C_X(t) dt$ and reaches its finite limit $(1/\rho) C_X(0, \bar{X})$ as $t \rightarrow \infty$, where \bar{X} is determined from the necessary condition (5) for $t \rightarrow \infty$, that is

$$\lim_{t \rightarrow \infty} p_t = \dot{p} = C_x(0, \bar{X}) + (1/\rho) C_X(0, \bar{X}). \quad (19)$$

It is interesting in passing to note how rigorously \bar{X} differentiates the economic concept of resource availability from its geological concept. \bar{X} is the *maximum* total amount of the resource that can optimally (economically) be exploited, given the expectations of prices and the state of technology.⁷ This is seen formally by noting from (5) that on an optimal path

$$dX_t = (1/C_{xx}) (-d\lambda_t - C_{xx} dx_t + dp_t - C_{xz} dz_t).$$

So, given current expectations of the state of technology and prices (i.e. $dz_t = dp_t = 0$), X_t obtains its maximum when $x_t \rightarrow 0$ and $|\lambda_t|$ reaches its finite limit, $(1/\rho) C_X(0, \bar{X})$, which is exactly what the condition (19) implies. Obviously, this maximum economic total will at any time be larger the higher the expected price levels and the greater the cost-reducing effect of technological change.

⁶ It should be noted that although $\dot{x} \geq 0$ is sufficient for this to happen, it is by no means a necessary condition since $\dot{C}_X > 0$ as long as $\dot{x}_t/x_t > -C_{xx}/C_{xx}$.

⁷ This accords with the concept of 'ultimate recoverable resource' which is often cited as the economically relevant measure of resource availability for minerals and fossil fuels. See, for example, Nordhaus (1974).

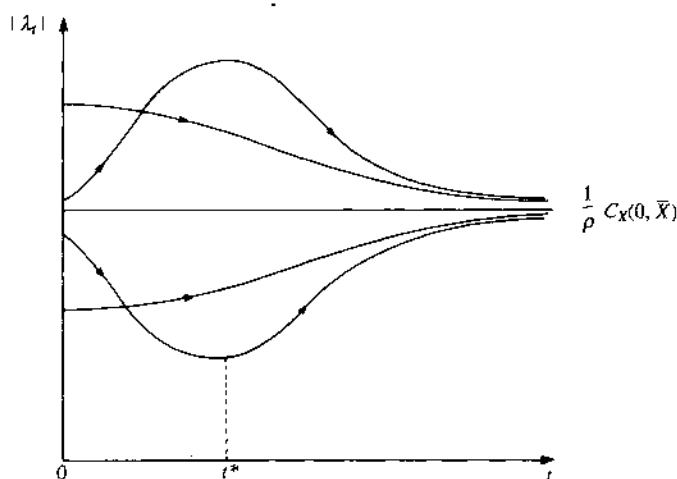


Fig. 1. Time paths of scarcity rent.

Case (b): $C_X(t)$ decreases monotonically over time, i.e. $\dot{C}_X(t) < 0$. In this case, which is the symmetry of Case (a), the shadow scarcity rent $|\lambda_t|$ will according to (14) monotonically decrease along an optimal path until at the limit, as $t \rightarrow \infty$, attains its finite limit $(1/\rho) C_X(0, \bar{X}) > 0$. As in Case (a), \bar{X} is determined from condition (19).

In between the foregoing two cases is the rather interesting special situation in which the total extraction cost increases linearly with the cumulative extraction according to $C(x_t, X_t) = kX_t + \gamma(x_t)$, where $k > 0$, $\gamma(0) = \gamma_x(0) = 0$, and $C_{xx} = \gamma_{xx} > 0$. Since $C_X(t) = k$ for all $t \geq 0$, it is readily verified from (9) that the shadow scarcity rent remains constant over time at $|\lambda_t| = k/\rho$. Thus, for this case, the externality cost of cumulative extraction (or depletion effect) is accounted for simply by adding a fixed surcharge k/ρ to the marginal extraction cost in every period. One can then treat the resource like a conventional reproducible product and set its augmented marginal cost equal to price to determine the optimal extraction rate; that is, from (5)

$$p_t = \gamma_x(x_t) + k/\rho, \quad (20)$$

where it is noted that so long as the resource price exceeds the fixed surcharge ($p_t > k/\rho$) the resource will continue to be supplied on a path characterised by $\dot{x}_t = \dot{p}/\gamma_{xx}$.

Case (c): $C_X(t)$ takes any permissible non-monotonic form. For any such functional form there always exists at least one $\tau = \tau^* > 0$ such that $(d/d\tau) C_X(\tau^*) = 0$ and $(d/d\tau) C_X(\tau) \leq 0$ in some interval of τ^* . Then, by (14) and (17), there will generally exist some $t = t^*$ for which $|\lambda_t| = 0$, implying that in general $|\lambda_t|$ will be non-monotonic.

Fig. 1 depicts possible paths of $|\lambda_t|$ corresponding to different monotonic and non-monotonic forms of $C_X(t)$.⁸

⁸ A variety of situations may cause $C_X(t)$ to be a non-monotonic function of time even in the absence of technological change. For example, it is plausible that during early periods $C_X(t)$ remains small because x_t and hence X_t stay at low levels, but later on increases as both x_t and X_t become large, and, after reaching a

The direct correspondence between the path of $|\lambda_t|$ and the shape of $C_X(t)$ can be illustrated by an example. Let $C_X(t)$ be presented by a polynomial of third degree in time: $C_X(t) = a + bt + ct^2 + dt^3$ where a, b, c , and d take values for which $C_X(t) > 0$ for all $t > 0$. Substituting this in (9) and integrating yields a path for $|\lambda_t|$ which is also a polynomial of the same degree given by $|\lambda_t| = \alpha + \beta t + \gamma t^2 + \delta t^3$, where

$$\alpha = a/\rho + b/\rho^2 + 2c/\rho^3 + 6d/\rho^4; \quad \beta = b/\rho + 2c/\rho^2 + 6d/\rho^3 \\ \gamma = c/\rho + 3d/\rho^2; \quad \text{and} \quad \delta = d/\rho.$$

Noting that $|\dot{\lambda}_t| = \beta + 2\gamma t + 3\delta t^2$, it is readily checked that for $\delta > 0$, $\beta > 0$, and $\gamma < -\sqrt{(3\beta\delta)}$, the scarcity rent first increases to a maximum at $t_1 = [-\gamma - (\gamma^2 - 3\beta\delta)^{1/2}]/3\delta$ and then declines to a minimum at $t_2 = [-\gamma + (\gamma^2 - 3\beta\delta)^{1/2}]/3\delta$ (the reverse path prevails for $\delta < 0$, $\beta < 0$, and $\gamma > \sqrt{(3\beta\delta)}$). In the simpler, quadratic, case where $d = 0$ the scarcity rent also follows a quadratic path $|\lambda_t| = (a/\rho + b/\rho^2 + 2c/\rho^3) + (b/\rho + 2c/\rho^2)t + c/\rho t^2$ which, for $c < 0$ and $b < -2c/\rho$, first decreases over the interval $(0, -(b/2c + 1/\rho))$ and then increases thereafter.

II. SPECIAL CASES, EMPIRICAL RESULTS, AND NON-MONOTONIC SCARCITY RENT PATH

In this section, I first show that the theoretical views in the literature regarding the time path of scarcity rent, being based on highly restrictive assumptions, hold only as *special* cases of the general result obtained in the preceding section and that in some cases they do not necessarily hold. I then argue that the hypothesis of non-monotonic scarcity rent path forwarded here is general enough to accommodate the empirical findings that have defied the conventional views of the dynamics of scarcity rent or have contrasted each other.

II.A. *Special cases in the literature*

I begin by noting that the extraction cost functions analysed by Heal (1976), Hanson (1980), and Solow and Wan (1976) are assumed to be constant-returns-to-scale of the form $C(x_t, X_t) = x_t g(X_t)$. Accordingly, the unit (= marginal) cost of extraction $g(X_t)$ is assumed to be an increasing function of cumulative extraction ($g' > 0$), but independent of the current rate of extraction x_t . More stringently, both Heal and Hanson assume the unit extraction cost to be bounded above by the constant, and relatively high, unit cost of a backstop substitute to which a switch is made once the stock \bar{X} of low-cost deposits are exhausted, i.e. $g(X_t) = \beta$ for $X_t \geq \bar{X}$. Under these assumptions it is easily verified that

$$C_X(x_t, X_t) = x_t g'(X_t) = g'(X_t) \dot{X}_t = (d/dt) g(X_t), \quad (21)$$

maximum, declines as x_t begins to fall rapidly and X_t grows sluggishly (see (15)). An alternative possibility is when the extraction rate during early years of resource life remains at a plateau rate (i.e. $\dot{x}_t = 0$) determined by the initial extraction capacity installed, in which case C_X first rises with time and subsequently falls as extraction rate rapidly declines.

so that by (13)

$$|\lambda_t| = \int_t^\infty e^{-\rho(\tau-t)} g'(X_\tau) \dot{X}_\tau d\tau. \quad (22)$$

Changing the variable of integration from τ to X_τ ,

$$|\lambda(X_t)| = \int_{X_t}^\infty e^{-\rho(\tau-t)} g'(X_\tau) dX_\tau. \quad (23)$$

But, since $g'(X_\tau) = 0$ for $X_\tau \geq \bar{X}$, it follows that

$$\lim_{X_t \rightarrow \bar{X}} |\lambda(X_t)| = 0. \quad (24)$$

This brings out vividly the crucial part that the assumption of backstop substitute (and more generally the boundedness of the unit cost function) plays in these special cases to reach the conclusion that the scarcity rent vanishes as the stock of low-cost resources is exhausted.

Furthermore, Hanson's (1980) proposition that if the unit extraction cost $g(X_t)$ is a concave function of time then the scarcity rent path decreases monotonically is also seen to follow immediately as a special case of the general result derived here, since for that case $(d/dt) C_X(t) = (d^2/dt^2) g(X_t) < 0$ so that (by (14)) $|\dot{\lambda}_t| < 0$.

Hanson, referring to Brobst (1979), attributes the concavity (in t) of $g(X_t)$ to the premise that 'resources with higher extraction costs are relatively more abundant'. However, even if this presumption were to be granted, it would imply the concavity of $g(X_t)$ in X_t and not necessarily in t , so that the scarcity rent path need not be necessarily decreasing monotonically. To see this, simply note that $(d^2/dt^2) g(X_t) = g''(X_t) \dot{X}_t^2 + g'(X_t) \ddot{X}_t$, which need not be negative even if $g''(X_t) < 0$.

Moreover, as Brobst notes from Skinner's (1976) study, for abundant minerals the amount available is expected to be *lognormally* distributed as a function of grade which 'suggests that the amount of metal available at a given grade will increase geometrically down to a grade corresponding to the peak of the curve and then decline in both grade and tonnage' (p. 125). If we accept this tonnage-grade distribution and the commonly accepted economic propositions that (a) higher grade resources are less costly to exploit, and (b) on an optimal (competitive) path higher grade deposits are depleted first, it will then follow that the unit extraction cost will first be concave (i.e. $(d^2/dt^2) g(X_t) = (d/dt) C_X(t) < 0$) for an initial period (as in Hanson's proposition) but *convex* ($(d^2/dt^2) g(X_t) = (d/dt) C_X(t) > 0$) thereafter; implying (by (14)) that even in the special case analysed by Hanson, the scarcity rent path $|\lambda_t|$ can well be non-monotonic.

Heal, contrary to Hanson, assumes $g(X)$ to be convex in X ($g''(X) > 0$), and yet like Hanson he concludes that the scarcity rent declines monotonically over time! The general result presented here resolves this theoretical disaccord by noting that although $|\lambda_t| \rightarrow 0$ as $X_t \rightarrow \bar{X}$, it *need not* be monotonically declining as Heal has maintained. The reason is simple: given that in his case $g''(X) > 0$,

$(d/dt) C_X(t) = g'(X_t) \dot{x}_t + g''(X_t) x_t^2$ need not be negative, even if $\dot{x}_t < 0$. In fact, as noted earlier, it is quite plausible that initially $\dot{x}_t/x_t > -g''(X_t) x_t/g'(X_t)$ (or even $\dot{x}_t \geq 0$) so that $(d/dt) C_X(t) > 0$ for an initial period, thereby giving rise (by (14)) to a situation in which $|\lambda_t|$ rises first to a maximum before it declines and eventually vanishes.

It is therefore seen that, even as a special case, the conclusion in the literature that the scarcity rent declines monotonically over time requires for its validity extremely restrictive assumptions about resource availability and extraction cost behaviour.

Turning to the case analysed by Solow and Wan (1976), their cost function is essentially the same as that assumed both by Heal (1976) and Hanson (1980), namely the unit extraction cost increases as a function of cumulative extraction alone. However, in contrast to those cases, there is no backstop substitute with fixed unit cost. It is then quite surprising that despite this rather marked difference in assumptions about the unit cost, Solow and Wan also conclude that the scarcity rent, $|\lambda_t|$, should monotonically fall to zero. What underlies their conclusion and is particularly special about their case is the application of Rawls' maxi-min criterion to the intertemporal resource allocation problem, and therefore the assumption that *discount rate is zero*. In fact, one can state the following general proposition:

PROPOSITION. *In the special case where $\rho = 0$ the scarcity rent always falls monotonically to zero regardless of the functional form of $C_X(t)$.*

Proof. For $\rho = 0$, (13) reduces to $|\lambda_t| = \int_t^\infty C_X(\tau) d\tau$. Since $C_X(\tau) > 0$ for $\tau \geq 0$, it follows that $\lim_{t \rightarrow \infty} |\lambda_t| = 0$. Also, differentiating with respect to t yields $\dot{|\lambda_t|} = -C_X(t) < 0$, for all $t \geq 0$.

That in these special cases the scarcity rent falls to zero is akin to economic intuition. In the cases analysed by Heal and Hanson, the stock of low-cost resource shrinks over time until at a certain date it is exhausted and the backstop substitute, with fixed unit cost, takes over. So, from that date onwards the cumulative amount extracted becomes irrelevant to the cost of obtaining a unit of resource (i.e. $C_X = 0$). In the case studied by Solow and Wan, an additional unit of resource extracted at any time t renders all future extractions slightly more costly (exactly by $g'(\tau) > 0$ per unit). In the absence of discounting, the sum total of all future additional costs ($\int_t^\infty g'(\tau) d\tau$) constitutes the scarcity rent attributable to date t . As lower-cost deposits are depleted over time, there remains simply a smaller amount of economically recoverable resources for which the unit extraction costs are pushed up by current extractions. So, the scarcity rent decreases over time until it vanishes altogether when all deposits of economically exploitable grades are exhausted.

This brings us to the work of Levhari and Livitan (1977) who have also analysed a model in which extraction cost rises with both the extraction rate and the cumulative amount already extracted. Labelling the scarcity rent as 'marginal profit' and assuming the stock of resource is fixed, the extraction horizon is finite, and exhaustion is incomplete, they conclude not only that the scarcity rent at the terminal date is zero, but *incorrectly* that 'globally the

marginal profit is a decreasing function of time. In this case the rate of growth of marginal profit is not only smaller than r but in fact negative (globally)⁹ (p. 185). That in this case the scarcity rent at the terminal date should be zero is a natural consequence of the assumption of incomplete exhaustion and follows immediately from the transversality condition for this special case. However, contrary to the authors' reasoning, the fact that the scarcity rent will be greater than zero at all times prior to the terminal date does *not* imply that its time path should necessarily be monotonically decreasing, thus rendering their conclusion generally invalid.⁹

Finally, coming to the model analysed by Kay and Mirrlees (1975), they postulate, as in the present paper, a cost function of the form $C(x_t, X_t)$ and an infinite extraction horizon, but, unlike the present model, a fixed resource stock. They maintain, in line with Hotelling rule, that along an optimal path the scarcity rent *grows monotonically* as the resource stock is depleted. However, this conclusion fails to note that since the time path of $|\lambda_t|$ depends crucially on that of $C_x(t)$ and since $C_x(t)$ need not be necessarily rising monotonically along an optimal extraction path, $|\lambda_t|$ can in general be a non-monotonic function of time (as shown here), and in special cases it can be constant or even declining monotonically over time.

II.B. *Empirical results in the literature*

The foregoing scrutiny reveals that only in highly special cases might one expect the time path of scarcity rent to be monotonic, and that in general it is non-monotonic. I shall now argue that the non-monotonicity hypothesis is general enough to embrace the empirical results that seem to defy the conventional theory or appear to be in conflict with each other. For one thing, it implies (by (10)) that *the competitive resource price path can be non-monotonic even if the marginal extraction cost $C_x(t)$ happens to be monotonic throughout time*. Accordingly, a monotonic resource price path (as predicted, for example, by Hotelling rule and its variations) should be considered more as an exception than rule. Empirical evidence supporting non-monotonicity of price path and, indirectly, of scarcity rent path, include: (a) Smith's (1979a) finding of considerable instability, over the 1900–73 period, in the estimated coefficients of simple *linear* time trends of relative resource prices which were postulated and supported by Barnett (1979); (b) Hall and Hall's (1984) conclusion that for mineral fuels the trends in relative prices shifted significantly from declining during the 1960s to rising during the 1970s, and (c) Slade's (1982) obtaining strong statistical evidence in support of a quadratic, U-shaped (versus linear) time path of real prices for eleven major metals and fuels over the period 1870 to 1978.¹⁰

⁹ As was just noted in the discussion of Heal's model, it is quite possible that the scarcity rent rises during an initial period and then falls as the terminal date is approached.

¹⁰ Heal and Barrow (1980) and Agbeyegbe (1989) provide empirical evidence showing that for several metals price movements have been strongly influenced not by the level of real interest rate – as the conventional theory would suggest – but by *changes* in it. Accordingly, possible oscillations in the real interest rate can induce non-monotonic resource price path. It should however be noted that in the present study, it is not the interest rate variation that causes non-monotonicity – as the interest rate is assumed here to remain constant – but rather the behaviour of the incremental cost of cumulative extraction, $C_x(x_t, X_t, z_t)$. In fact,

Clearly, the non-monotonicity hypothesis can be consistent with observations of rising, or, as in Nordhaus (1974), declining trends in resource prices as well as with rising or declining trends in scarcity rent over a *specific* period of time. But, it warns us against extrapolating any of such observations to deduce monotonic behaviour *throughout* time for these indicators.

Furthermore, as seen from (9), at any time t , it is the entire future path of the incremental cost of cumulative extraction $\{C_x(\tau); \tau \geq t\}$, and not the current marginal extraction cost $C_x(t)$, that determines (for a given ρ) the magnitude of the scarcity rent. So, *the path of scarcity rent need not have any particular relationship with movements in the marginal (or unit) extraction cost*. This theoretical implication accords well with Brown and Field's (1978) observation that in the United States between 1940 and 1970 the scarcity rent for timber rose sharply while its unit cost steadily declined. It also agrees with Devarajan and Fisher's (1982) finding that the scarcity rent for US oil and gas, as approximated by average discovery costs, witnessed a rising trend over the 1946–71 period while the unit extraction cost tended to decline. Viewed from this perspective, Barnett and Morse's observation of declining trends in the unit extraction costs of minerals during the 1870–1957 period should not be misinterpreted (as often is) to be necessarily rejecting the possibility of rising trends in scarcity rents either over the same period or in subsequent periods.

Direct empirical support for our hypothesis of non-monotonic scarcity rent path comes from Pesaran's (1990) estimate of the shadow scarcity rent for crude oil in the UK Continental Shelf. Based on an exploration function estimated from quarterly data for the 1978–86 period, he obtains an estimate of the path of scarcity rent that shows a strongly non-monotonic behaviour, with an average trend that declines over the 1978–9 period, rises during 1980–2, and remains fairly constant thereafter. Further empirical evidence supporting the non-monotonicity hypothesis is provided by Lasserre and Ouellette (1991) who use a modified translog cost function and estimate C_x for Canadian asbestos over the 1953–85 period. Their estimate of C_x indicates a long-run non-monotonic trend with negative values during two periods: 1974–6 and 1977–82. They attribute the negative values to the difficulty of adjusting mineral reserve evaluation to changes in economic conditions.¹¹

since Heal and Barrow find strong statistical support for the summing-to-zero of the coefficients of the (current and lagged) interest rates, in their study, constancy of the interest rate would imply a *stationary* resource price path.

¹¹ Halvorsen and Smith (1984) also use a translog cost function to derive the shadow scarcity rent for the output of the Canadian metal mining industry. Their estimates of the shadow scarcity rent show a declining trend from 1956 to 1974, but they are derived from an estimated cost function that fails to include the stock depletion effect (C_x), which the present analysis has shown to be the key cause of non-monotonic scarcity rent path. Farrow (1985) also estimates a translog cost function for a confidential firm in a rock mining industry. Using the estimates of marginal extraction cost and the stock effect together with an estimated price expectations formation scheme he derives estimates of scarcity rent which show a declining trend over the 1975–81 period. However, his result is based on several stringent assumptions, especially those of constant ore grade at all mining depths and strictly increasing mine depth, which rule out the possibility of non-monotonic stock depletion effect $C_x(t)$.

III. TECHNOLOGICAL CHANGE AND SCARCITY RENT PATH

Thus far, to conform with premises of the studies reviewed in the previous section, the analysis has focused on non-monotonic behaviour of scarcity rent in the absence of technological change.¹² To bring out the effect of technological change on the time path of scarcity rent, substitute from (15) in (14) to write:

$$|\dot{\lambda}_t| = \int_t^\infty e^{-\rho(\tau-t)} C_X(\tau) \left[\xi_{Xx}(\tau) \frac{\dot{x}}{x} + \xi_{Xx}(\tau) \frac{\dot{X}}{X} - \xi_{Xz}(\tau) \frac{\dot{z}}{z} \right] d\tau, \quad (25)$$

where

$$\xi_{Xx} = \frac{x C_{Xx}}{C_X}, \quad \xi_{Xx} = \frac{X C_{Xx}}{C_X} \quad \text{and} \quad \xi_{Xz} = -\frac{z C_{Xz}}{C_X} > 0$$

are the elasticities of $C_X(x_\tau, X_\tau, z_\tau)$ with respect to x_τ , X_τ , and z_τ , and in general vary with time.

It is apparent from (25) that, by countervailing the effects of accelerated extraction rate and cumulative depletion in raising the incremental cost C_X , technological change can further contribute to non-monotonicity of the scarcity rent path. To make the point sharply, let us make the restrictive assumptions that (a) the elasticities ξ_{Xx} , ξ_{Xx} and ξ_{Xz} are constants, and (b) the rates of change of x_τ , X_τ and z_τ are all linear functions of time:

$$\frac{\dot{x}}{x} = k_1 + a\tau, \quad \frac{\dot{X}}{X} = k_2 + b\tau \quad \text{and} \quad \frac{\dot{z}}{z} = k_3 + c\tau,$$

where k_2 , k_3 , b and c are non-negative to ensure $\dot{X}/X > 0$ and $\dot{z}/z > 0$ for all $\tau \geq 0$. Substituting these in (25), performing the integration and using (9), yields:

$$|\dot{\lambda}_t| = (A + Bt)|\lambda_t| - B \frac{\partial |\lambda_t|}{\partial \rho}, \quad (26)$$

where

$$A = k_1 \xi_{Xx} + k_2 \xi_{Xx} - k_3 \xi_{Xz}, \quad B = a \xi_{Xx} + b \xi_{Xx} - c \xi_{Xz},$$

and where

$$\frac{\partial |\lambda_t|}{\partial \rho} = - \int_t^\infty (\tau - t) e^{-\rho(\tau-t)} C_X(\tau) d\tau < 0$$

accords with the conventional wisdom that, for $\rho > 0$, the scarcity rent at any time t varies inversely with the discount rate.¹³

It is clear from (26) that even the restrictive assumptions made above are not sufficient to ensure a monotonic path for $|\lambda_t|$. The additional assumptions needed for that purpose would be either (i) $A > 0$ and $B > 0$, or (ii) $A < 0$ and

¹² To be sure, Slade (1982) incorporates the effect of technological change in a highly simplified cost function to suggest a U-shaped time path for the relative resource price. She does not however analyse the effect of technological change on the time path of scarcity rent. In fact, her suggested time path for the resource price rests on the presumption that the resource rent monotonically increases with time.

¹³ As Farzin (1984, 1986) has shown, where there are capital costs involved in resource extraction or in production of a substitute, $|\lambda_t|$ can also be a non-monotonic function of ρ .

$B < 0$, or (iii) $A = 0$, which as a special case would also hold when $k_1 = k_2 = k_3 = 0$, or (iv) $B = 0$, which would require the constancy of the *net* rate of change in C_x due to changes in the extraction rate, cumulative extraction and technological change, and would also hold as a special case when $a = b = c = 0$, i.e. when x , X , and z , change at constant proportional rates of k_1 , k_2 and k_3 , respectively.

Of course, realistically, technological change is neither a predictable nor a steady, smooth phenomenon. Rather, it can be more plausibly characterised as sequences of time spells where a period of technological abeyance ($\dot{z} = 0$) is followed by an interval of progress ($\dot{z} > 0$) and where both the timing and duration of each interval are unpredictable. Such a discontinuous and unpredictable pattern of technological change can further reinforce the non-monotonicity of scarcity rent path, for then it will be even more likely that increases in incremental cost $C_x(t)$ cease during an interval when technological progress counterbalances the cumulative depletion effect (so that $\dot{C}_x \leq 0$), but resume again during another period when the technology is in abeyance and the depletion effect dominates (so that $\dot{C}_x > 0$). A similar effect can also result from unanticipated additions to reserves due to new discoveries.¹⁴

IV. CONCLUDING REMARKS

The theoretical views on the dynamics of scarcity rent for an exhaustible resource have been sharply opposing: while some authors have in keeping with the conventional Hotelling rule maintained that the scarcity rent should continually grow as a resource is depleted, others have argued exactly the reverse. The purpose of the present paper has been to shed a new light on this controversy toward its resolution. By analysing the dynamics of scarcity rent under competitive conditions with an extraction cost function that explicitly allows not only for the effect of extraction rate but also those of cumulative extraction and technological change, it has shown that in general the time path of scarcity rent is *non-monotonic*, so that the views in the literature are valid only as special cases. It has also argued that the hypothesis of non-monotonic scarcity rent is sufficiently general to embrace the empirical findings that have been in defiance of the conventional wisdom on the dynamics of scarcity rent and in conflict with each other.

In addition to synthesising the literature results on the dynamics of resource scarcity, the hypothesis of non-monotonic scarcity rent can provide a partial explanation for the commonly observed oscillatory behaviour of resource rents over long periods of time. More importantly, it has strong policy implications particularly for the resource-based developing countries. It underlines their economic vulnerability to possibility of large swings in their principal sources of foreign exchange earnings. The difficulty of predicting the rent swings and the attendant economic uncertainties warn economic planners not to base long-term investment decisions on extrapolations of rent incomes during a transitory

¹⁴ For the effect of unanticipated discoveries on the resource rent see particularly Lasserre (1985).

phase (especially upswing) of the oscillatory long-term path. Otherwise, heavy losses may be incurred either in the form of foregone economic opportunities or macroeconomic adjustment costs (when the rent cycle takes a downswing). It would therefore be too risky and unsound to rely on predictions of a monotonic time path of resource rent from highly specialised models, whether they predict steadily rising rents, like Hotelling-type models, or steadily declining rents, as in some of the special models reviewed in this paper. One such example of neglecting the non-monotonicity of scarcity rent path by relying on the Hotelling rule is Boskin *et al.*s (1985) evaluation of US federal oil and gas mineral rights. Assuming that the real oil price received by US producers would steadily increase at the real rate of interest (taken to be 3% annually) and ignoring exploration, development and extraction costs, they estimated the value of federal government's oil and gas holdings in 1981 to be \$819 billion, which they noted to exceed the value of the privately held national debt (\$794 billion) in that year. The danger of such euphoric calculations is best indicated by Adelman's (1990, p. 1) observation that 'On their assumptions, 1981 Mexican oil reserves were worth nearly \$2,000 billion, and it was ultra-conservative to borrow only \$60 billion against them'.

Clearly, where the resource rent accrues directly to the government (as is the case in most developing countries), its oscillatory behaviour over time necessitates devising a stabilising scheme for its optimal management and use. What characteristics such a scheme should have is a question for future research.

Also, underlying the non-monotonic scarcity rent hypothesis in this paper has been the behaviour of the incremental cost of cumulative extraction over time, $C_x(t)$, and particularly the assumptions about how it may vary with changes in the extraction rate ($C_{xx} > 0$), cumulative depletion ($C_{xx} > 0$), and technology ($C_{xz} < 0$). What seems, next, to be of great value is empirical research into these effects for a large variety of resources and over wide ranges of time periods.

Finally, in this paper the dynamics of scarcity rent has been examined under competitive conditions and in the absence of exploration activities. Clearly, both non-competitive market structures and firms' exploration decisions can affect the time path of scarcity rent. While further research needs to be devoted to a thorough investigation of these effects, I conjecture that they very likely will confirm the non-monotonicity of scarcity rent path.

The American University, Washington D.C.

Date of receipt of final typescript: October 1991

APPENDIX

Existence of an Optimal Solution

The question of existence of solution in optimal control problems with infinite time horizon can be a notoriously difficult one. For this reason, in the economic literature on exhaustible resources it is usually *assumed* implicitly that a solution path exists.

Fortunately, in the present case, one can establish the existence of an optimal solution by resorting to an existence theorem essentially due to Baum (1976) (see specially *Theorem 7.1*, p. 114) and further modified and extended for economic problems by Seierstad and Sydsaeter (1987) (see *Theorem 15*, p. 237).

In the case of present problem with a single control variable $x(t)$ and state variable $X(t)$, the Seierstad and Sydsaeter's *Theorem 15* simplifies into the following special case.

THEOREM. Consider the problem

$$\text{Maximise}_{x_t} \int_0^\infty e^{-\rho t} [P_t x_t - C(x_t, X_t, z_t)] dt,$$

$$\text{where} \quad \dot{X}_t = x_t \geq 0, \quad X(0) = 0, \quad (\text{A } 1)$$

$$x_t \geq 0 \text{ is continuous on } [0, \infty), \quad (\text{A } 2)$$

$$x_t \in U, \quad U \text{ a fixed, non-negative subset of } R^1, \quad (\text{A } 3)$$

and where

$$f_0(x, X, t) \equiv e^{-\rho t} [P_t x_t - C(x_t, X_t, z_t)] \quad \text{and} \quad f(x, X, t) \equiv \dot{X}_t = x_t \text{ are continuous.}$$

A pair (x_t, X_t) is admissible if it satisfies (A 1)–(A 3) for all $t \in [0, \infty)$. Suppose that U is closed and bounded and that there exist piecewise continuous functions $\phi_i \geq 0$, $i = 0, 1$ with $\int_0^\infty \phi_i(t) dt < \infty$ such that

$$f_0(x, X, t) \leq \phi_0(t), \quad (\text{A } 4a)$$

$$\dot{f}(x, X, t) = \dot{X}_t = x_t \leq \phi_1(t) \quad (\text{A } 4b)$$

for all admissible pairs (X_t, x_t) and all $t \in [0, \infty)$. Suppose further that there exist piecewise continuous, non-negative functions $a(t)$ and $b(t)$ such that

$$\dot{X} = x_t \leq a(t) X_t + b(t). \quad (\text{A } 5)$$

Assume finally that the set

$$N(X, x, t) = \{f_0(X, x, t) + \gamma, x: x \in U, \gamma \leq 0\} \quad (\text{A } 6)$$

is convex for all (X, t) . Then the existence of an admissible pair in the problem implies the existence of an optimal pair (X^*, x^*) . ■

I now show that all the conditions of the existence Theorem are satisfied in the special case of present problem.

Condition (A 6). For our problem the convexity of the set $N(X, x, t)$ is directly implied by the assumption of convexity of $C(x, X)$ in (x, X) . To see this, simply note that $f_0(x, X, t)$, and *a fortiori* $f_0(x, X, t) + \gamma$, is a concave function of x_t for all (X, t) if $C_{xx} > 0$.

Condition (A 5). Since in the present case $X_t = \int_0^t x_\tau d\tau$ and $x_\tau \geq 0$ for all $\tau \geq 0$, it follows trivially that for $a(t) = 1$ and any $b(t) \geq 0$ condition (A 5) is always satisfied.

Condition (A 4). Condition (A 4a) is satisfied for our problem because so long as the profit function $\Pi(t)$ is bounded, that is

$$0 \leq \Pi(t) \equiv [P_t x_t - C(x_t, X_t, z_t)] \leq K < \infty$$

for all admissible (x_t, X_t) and all $t \in [0, \infty)$, then for any $\rho > 0$ one can choose $0 \leq \phi_0(t) = K e^{-\rho t}$ such that $\int_0^\infty \phi_0(t) dt = K/\rho < \infty$ and

$$f_0(x, X, t) = \Pi(t) e^{-\rho t} \leq \phi_0(t) = K e^{-\rho t}.$$

Also, as an implication of (A 4b) one should have $X(\infty) = \int_0^\infty x_t dt \leq \int_0^\infty \phi_1(t) dt < \infty$. This condition is naturally satisfied in the present case since the convexity of $C(x, X)$

in X implies that $C_X \rightarrow \infty$ as $X \rightarrow \infty$. As such, letting $X \rightarrow \infty$ defies economic sense. Now, for any admissible x_t , one can define $\phi_1(t) = x_t + \delta e^{-Kt}$ for any arbitrary small $\delta \geq 0$ and any arbitrary large K such that it satisfies (A 4a) and

$$\int_0^{\infty} \phi_1(t) dt = \int_0^{\infty} x_t dt + \frac{\delta}{K} < \infty.$$

This completes the proof of the existence of an optimal solution pair (X_t^*, x_t^*) . Moreover, as is well-known, the solution path $x^*(t)$ is unique if $f_0(X, x, t)$ is strictly concave in x for each (X, t) , which is the case here because of the assumption of convexity of $C(x, X)$ in (x, X) .

REFERENCES

- Adelman, M. A. (1990). 'Mineral depletion, with special reference to petroleum.' *Review of Economics and Statistics*, vol. 1 (February), pp. 1-10.
- Agbeyegbe, T. (1989). 'Interest rates and metal price movements: further evidence.' *Journal of Environmental Economics and Management*, vol. 16, pp. 184-92.
- Barnett, H. J. (1979). 'Scarcity and Growth Revisited.' In Smith (1979b), pp. 163-217.
- and Morse, C. (1963). *Scarcity and Growth: The Economics of Natural Resource Availability*. Baltimore: Johns Hopkins University Press (for Resources for the Future).
- Baum, R. F. (1976). 'Existence theorems for Lagrange control problems with unbounded time domain.' *Journal of Optimization Theory and Applications*, vol. 19, pp. 89-116.
- Boskin, M., Robinson, M. S., O'Reilly, T. and Kumar, P. (1985). 'New estimates of the value of federal mineral rights and land.' *American Economic Review*, vol. 75 (December), pp. 923-36.
- Brobst, D. A. (1979). 'Fundamental concepts for the analysis of resource availability.' In Smith (1979b), pp. 106-42.
- Brown, G. M., Jr. and Field, B. C. (1978). 'Implications of alternative measures of natural resource scarcity.' *Journal of Political Economy*, vol. 86, no. 2, pt. 1 (April), pp. 229-43.
- Devarajan, S. and Fisher, A. C. (1982). 'Exploration and scarcity.' *Journal of Political Economy*, vol. 90 (December), pp. 1279-90.
- Farrow, S. (1985). 'Testing the efficiency of extraction from a stock resource.' *Journal of Political Economy*, vol. 93 (June), pp. 452-87.
- Farzin, Y. H. (1984). 'The effect of discount rate on depletion of exhaustible resources.' *Journal of Political Economy*, vol. 92 (October), pp. 841-51.
- (1986). *Competition in the Market for an Exhaustible Resource*. Connecticut: JAI Press Inc.
- Fisher, A. C. (1979). 'Measures of Natural Resource Scarcity.' In Smith (1979b), pp. 249-75.
- (1981). *Resource and Environmental Economics*. Cambridge: Cambridge University Press.
- Hall, D. C. and Hall, J. V. (1984). 'Concepts and measures of natural resource scarcity with a summary of recent trends.' *Journal of Environmental Economics and Management*, vol. 11, pp. 363-79.
- Halvorsen, R. and Smith, T. R. (1984). 'On measuring natural resource scarcity.' *Journal of Political Economy* (October), pp. 954-64.
- Hanson, D. A. (1980). 'Increasing extraction costs and resource prices: some further results.' *Bell Journal of Economics*, vol. 11 (Spring), pp. 335-42.
- Heal, G. M. (1976). 'The relationship between price and extraction cost for a resource with a backstop technology.' *Bell Journal of Economics*, vol. 7 (Autumn), pp. 371-8.
- and Barrow, M. (1980). 'The relationship between interest rates and metal price movements.' *Review of Economic Studies*, vol. 47, pp. 161-81.
- Hotelling, H. (1931). 'The economics of exhaustible resources.' *Journal of Political Economy*, vol. 39 (April), pp. 137-75.
- Johnson, M. H., Bell, F. W. and Bennett, J. T. (1980). 'Natural resource scarcity: empirical evidence and public policy.' *Journal of Environmental Economics and Management*, vol. 7 (September), pp. 256-71.
- Kay, J. and Mirrlees, J. A. (1975). 'The desirability of natural resource depletion.' In Pearce and Rose (1975), pp. 140-76.
- Lasserre, P. (1985). 'Discovery costs as a measure of rent.' *Canadian Journal of Economics*, vol. 18 (August), pp. 474-83.
- and Ouellette, P. (1991). 'The measurement of productivity and scarcity rents: the case of asbestos in Canada.' *Journal of Econometrics*, vol. 48, no. 3 (June), pp. 287-312.
- Levhari, D. and Livitan, N. (1977). 'Notes on Hotelling's economics of exhaustible resources.' *Canadian Journal of Economics*, vol. 10, no. 2 (May), pp. 177-92.
- Nordhaus, W. D. (1974). 'Resources as a constraint on growth.' *American Economic Review*, vol. 64 (May), pp. 22-6.

- Pearce, D. W. and Rose, J. (ed.) (1975). *The Economics of Natural Resource Depletion*. London: Macmillan.
- Pesaran, M. H. (1990). 'An econometric analysis of exploration and extraction of oil in the UK Continental Shelf.' *ECONOMIC JOURNAL*, vol. 100 (June), pp. 367-90.
- Pindyck, R. S. (1977). *Advances in the Economics of Energy and Resources*, vol. 2. Connecticut: JAI Press Inc.
- (1978). 'The optimal exploration and production of non-renewable resources.' *Journal of Political Economy*, vol. 86 (October), pp. 841-61.
- Seierstad, A. and Sydsaeter, K. (1987). *Optimal Control Theory with Economic Applications*. Amsterdam: North-Holland.
- Skinner, B. J. (1976). 'A second iron age ahead?' *American Scientist*, vol. 64 (May-June), pp. 258-69.
- Slade, M. E. (1982). 'Trends in natural-resource commodity prices: an analysis of the time domain.' *Journal of Environmental Economics and Management*, vol. 9 (June), pp. 122-37.
- Smith, V. K. (1979a). 'Natural resource scarcity: a statistical analysis.' *Review of Economics and Statistics*, vol. 61 (August), pp. 423-7.
- (1979b). *Scarcity and Growth Reconsidered*. Baltimore: Johns Hopkins University Press (for Resources for the Future).
- Solow, R. M. and Wan, F. Y. (1976). 'Extraction costs in the theory of exhaustible resources.' *Bell Journal of Economics*, vol. 7 (Autumn), pp. 359-70.
- Zimmerman, M. B. (1977). 'Estimating a policy model of US coal supply.' In Pindyck (1977), pp. 59-92.